Strong-coupling analysis of QED_3 for excitation spectrum broadening in the undoped high-temperature superconductors

T. Morinari*
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
(Received 1 October 2007; published 27 February 2008)

Theory of quantum electrodynamics in three spatial-time dimensions is applied to the two-dimensional $S=1/2$ quantum Heisenberg antiferromagnet in order to investigate a doped hole in high-temperature superconductors. Strong coupling analysis of the U(1) gauge field interaction is carried out to describe spectral broadening observed in the undoped compounds. It is found that the fermionic quasiparticle spectrum is of Gaussian form with the width about $3J$, with J being the superexchange interaction energy. The energy shift of the spectrum is on the order of the quasiparticle bandwidth, which suggests that the system is in the strong coupling regime with respect to the gauge field interaction describing the phase fluctuations about the π-flux state.

DOI: 10.1103/PhysRevB.77.075128 PACS number(s): 74.72.−h, 79.60.−i, 75.10.Jm, 11.10.Kk

I. INTRODUCTION

One of the most fundamental questions about high-temperature superconductivity is how to describe doped holes introduced in the CuO_2 plane. The simplest way to approach this problem is to investigate the single hole doped system. Experimentally, the excitation spectrum associated with a single hole is observed by angle resolved photoemission spectroscopy (ARPES) in the undoped compounds such as $Sr_2CuO_2Cl_2$ (Refs. 1 and 2) and $Ca_2CuO_2Cl_2$.[3] Although the spectrum is not sharp but quite broad, whose width is ranging from 0.1 to 0.5 eV, the experiments show that the trace of the peak indicates a dispersion whose maxima are at $(\pm\pi/2,\pm\pi/2)$. The dispersion near these points is quadratic and almost isotropic. The bandwidth is $2.2J\simeq 270$ meV with J the superexchange interaction. This bandwidth is much smaller than the band-structure estimation of $8t\simeq 2.8$ eV. Furthermore, the observed spectra are not described by a conventional Lorentzian form but described by a Gaussian form.[4] These observations suggest that the quasiparticle excitations in the undoped cuprates are quite different from conventional Fermi liquid quasiparticles.

In the slave-fermion theory of the t-J model, the single hole system has been analyzed by the self-consistent Born approximation.[5] The effect of the spin-wave excitations is included in the self-energy in a self-consistent manner with omitting vertex corrections. The resulting hole dispersion has minima at $(\pm\pi/2,\pm\pi/2)$, and the bandwidth is scaled by J. The dispersion along (0,0) to (π,π) is in good agreement with the experiments. Quantum Monte Carlo simulations based on a model, in which canonically transformed spinless fermions propagate with antiferromagnetic spin correlation background, such as the slave-fermion formalism, are consistent with this result.[6] However, the dispersion along $(\pi,0)$ to $(0,\pi)$ is much smaller than that in the experiments. This discrepancy is improved by including the next nearest neighbor and the third nearest neighbor hopping terms.[7] This suggests that within this approach the quadratic behavior near the $(\pi/2,\pi/2)$ point along $(0,\pi)$ to $(\pi,0)$ has a different origin from that along (0,0) to (π,π). Furthermore, it turns out that the damping effect due to the coupling with the spin-wave modes does not lead to a broad line shape.[8] Recently Mischenko and Nagaosa studied a coupling to an optical phonon.[9] They numerically summed over Feynman diagrams including vertex corrections for phonons. It was argued that the coupling is in the strong-coupling regime, and so the quasiparticle spectrum becomes broad. In this scenario the most enigmatic feature of the hole spectral broadening in the undoped compounds is associated with phonon effects. The dominant role is not played by the antiferromagnetic correlations which is believed to be essential for the mechanism of high-temperature superconductivity.

Here I take a different approach. I consider the π-flux state proposed in the literature[10] from a mean field theory of the $S=1/2$ antiferromagnetic Heisenberg model based on a fermionic representation of the spins. The dispersion of the quasiparticle in the π-flux phase is in good agreement with the experimentally obtained dispersion as pointed out by Laughlin.[11] Including phase fluctuations about the mean field, the effective theory is described by quantum electrodynamics in three spatial-time dimensions, which is called QED_3. At mean field level, the fermions are massless. By including the effect of the gauge field, the mass of the Dirac fermions is induced.[12] This mass is associated with the staggered magnetization. The presence of the mass term is also suggested by a variational Monte Carlo approach.[13] The quadratic dispersion around $(\pi/2,\pi/2)$ observed in the experiments is consistent with the massive quasiparticle spectrum. Furthermore, the quasiparticle dispersion is isotropic at $(\pi/2,\pi/2)$.

The purpose of this paper is to argue that the coupling of the fermions with the phase fluctuations leads to a broad Gaussian spectrum. The QED_3 action with the mass term is analyzed by performing a canonical transformation. The spectral function is obtained by calculating the Green's function of the Dirac fermion which is associated with a single quasiparticle propagation. It is shown that the spectral function shows a broad Gaussian peak whose width is on the order of J.

The rest of the paper is organized as follows. In Sec. II, we describe the QED_3 formalism of the π-flux state. After taking the transverse gauge, a canonical transformation is

 ©2008 The American Physical Society

applied. The quasiparticle Green's function is computed in Sec. III. The spectral function is obtained with including vertex functions arising from random phase approximation about the instantaneous longitudinal gauge field interaction. Implications of the result are discussed in Sec. IV.

II. QED$_3$ THEORY OF THE π-FLUX STATE

For the description of the $S=1/2$ two-dimensional quantum Heisenberg antiferromagnet, I take the following QED$_3$ action in the real time formalism as the effective theory:

$$S=\int d^3x\left\{\bar{\psi}(x)[i\gamma^\mu(\partial_\mu-ia_\mu)-m\sigma_3]\psi(x)-\frac{1}{4e_a^2}f_{\mu\nu}f^{\mu\nu}\right\}, \tag{2.1}$$

where the Dirac fermion fields, $\psi(x)$, consist of four components associated with even and odd sites and two independent nodes. Due to the spin degrees of freedom there are two species of $\psi(x)$. (For the derivation, see Appendix A.) Hereafter the spin index for $\psi(x)$ is suppressed because the spin degrees of freedom does not play an important role in the following calculation. The action (2.1) describes low-lying excitations around $(\pm\pi/2,\pm\pi/2)$ because the above continuum model was derived by taking the continuum limit at those points. Note that the theory is particle-hole symmetric. Therefore, the quasiparticle properties are identical to the quasihole properties.

For the gauge, I take the transverse gauge: $\nabla\cdot\mathbf{a}=0$. In this gauge, the interaction between the fermions mediated by the longitudinal part of the gauge field is instantaneous as in the conventional electromagnetic field formulation as follows:

$$\begin{aligned}S=&\int d^3x\bar{\psi}(x)[i\gamma^0\partial_t+i\gamma^j(\partial_j+ia_j)-m\sigma_3]\psi(x)\\&+\frac{e_a^2}{4\pi}\int d^3x\int d^2\mathbf{r}'[\rho(\mathbf{r},t)-\rho_0][\rho(\mathbf{r}',t)-\rho_0]\ln|\mathbf{r}-\mathbf{r}'|\\&+\frac{1}{2e_a^2}\int d^3x[(\partial_t\mathbf{a})^2-(\nabla\times\mathbf{a})^2],\end{aligned} \tag{2.2}$$

where the background gauge charge, $-e_a\rho_0$, comes from the constraint on the fermion number to represent the spin 1/2. In three spatial-time dimensions, the "Coulomb" interaction is described by $V(r)=-\frac{e_a^2}{2\pi}\ln r$. The presence of the log potential is associated with confinement of the Dirac fermions. Physically the confinement due to this log potential is associated with the mass gap of the Dirac fermions. This is understood from the derivation of the log potential: The dynamics of the gauge field is determined by the Dirac fermion bubble. In the presence of the Dirac fermion mass, we have a Maxwellian term which implies that the gauge field is massless and so it leads to the log potential. Therefore, the confinement is associated with that there is an energy gap to create a hole in the system. The gap is twice as large as the Dirac fermion mass. This is consistent with the experiments in which the excitation gap is clearly seen. [This energy gap appears at the end of the calculation, E_s, in Eq. (3.13).]

Under the transverse gauge, the vector potential is represented by

$$a_x(q)=-\frac{iq_y}{q}a(q),\quad a_y(q)=\frac{iq_x}{q}a(q), \tag{2.3}$$

in the momentum space. Quantizing the transverse gauge field, the Hamiltonian is

$$\begin{aligned}H=&\int d^2\mathbf{r}\bar{\psi}(\mathbf{r})(-i\gamma^j\partial_j+m\sigma_3)\psi(\mathbf{r})\\&+\int d^2\mathbf{r}\sum_q e^{iq\cdot\mathbf{r}}\bar{\psi}(\mathbf{r})\frac{i}{q}(q_x\gamma_y-\gamma_xq_y)\sqrt{\frac{e_a^2}{2\omega_q}}(b_q+b_{-q}^\dagger)\psi(\mathbf{r})\\&+\frac{1}{2}\int d^2\mathbf{r}\int d^2\mathbf{r}'V(|\mathbf{r}-\mathbf{r}'|)[\rho(\mathbf{r})-\rho_0][\rho(\mathbf{r}')-\rho_0]\\&+\sum_q\frac{\omega_q}{2}(b_qb_q^\dagger+b_q^\dagger b_q).\end{aligned} \tag{2.4}$$

In order to investigate the strong-coupling effects, I perform the following canonical transformation[14]:

$$\bar{H}=e^sHe^{-s}, \tag{2.5}$$

where

$$s=\int d^2\mathbf{r}\sum_q(b_q-b_{-q}^\dagger)\psi^\dagger(\mathbf{r})M_q(\mathbf{r})\psi(\mathbf{r}). \tag{2.6}$$

(The area of the system is set to unity.) This is a unitary transformation if $M_{-q}^\dagger(\mathbf{r})=M_q(\mathbf{r})$. The function $M_q(\mathbf{r})$ is chosen so that the interaction term between the Dirac fermions and the gauge field is canceled by $[s,H]$ as follows:

$$M_q(\mathbf{r})=-\sqrt{\frac{e_a^2}{2\omega_q^3}}e^{iq\cdot\mathbf{r}}\frac{i}{q}\gamma_0(q_x\gamma_y-\gamma_xq_y). \tag{2.7}$$

Under this canonical transformation, the fermion fields transform as

$$e^s\psi(\mathbf{r})e^{-s}=e^{X(\mathbf{r})}\psi(\mathbf{r}). \tag{2.8}$$

The function $X(\mathbf{r})$ is

$$X(\mathbf{r})=-\sum_q(b_q-b_{-q}^\dagger)M_q(\mathbf{r}). \tag{2.9}$$

Formally, this canonical transformation eliminates the vertex between the Dirac fermions and the gauge bosons, which diverges in the strong-coupling limit. (Note that a similar canonical transformation is discussed for a nonrelativistic fermion by Ng.[15])

III. QUASIPARTICLE GREEN'S FUNCTION

Now I compute the time-ordered Green's function

$$\begin{aligned}G(\mathbf{r},t)&=-i\langle T\psi(\mathbf{r},t)\psi^\dagger(0,0)\rangle\\&=-i\langle T\mathrm{e}^{X(\mathbf{r},t)}\psi(\mathbf{r},t)\psi^\dagger(0,0)\mathrm{e}^{-X(\mathbf{0},0)}\rangle_{\bar{H}}.\end{aligned} \tag{3.1}$$

This Green's function has a matrix form of 4×4. However,

the matrix is divided into two blocks in which each part describes either the Dirac fermion fields near $(\pi/2,\pi/2)$ or the Dirac fermion fields near $(-\pi/2,\pi/2)$. It is enough to focus on one of them because two components are decoupled as far as long-wavelength gauge field fluctuations are concerned. The superscript (1) is used to denote the former. Diagonalizing the factor with γ matrices, $X(\mathbf{r})$ is

$$X^{(1)}(\mathbf{r}) = i\sum_q (b_q e^{iq\cdot\mathbf{r}} + b_q^\dagger e^{-iq\cdot\mathbf{r}})\sqrt{\frac{e_a^2}{2\omega_q^3}}U_q\tau_3 U_q^\dagger, \quad (3.2)$$

where

$$U_q = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -(q_x - iq_y)/q \\ (q_x + iq_y)/q & 1 \end{pmatrix}. \quad (3.3)$$

The first term in Eq. (2.4), which does not change its form by the canonical transformation, is diagonalized by a unitary transformation as well. Finally, by making use of the following formula:

$$\langle e^{Ab^\dagger + Bb} e^{Cb^\dagger + Db}\rangle = e^{1/2(AB+CD+2BC)} e^{(A+C)(B+D)n(\omega)} \quad (3.4)$$

for bosons, Eq. (3.1) is

$$G^{(1)}(\mathbf{r},t) = -i\sum_k e^{i\mathbf{k}\cdot\mathbf{r}} e^{-iE_k t} R_k \exp[-K(\mathbf{r},t)], \quad (3.5)$$

at $T=0$. The retarded Green's function has the same form for $t>0$. Here,

$$R_k = \prod_{\mathbf{q}}\left[\frac{1}{4}\left(1 + \frac{k_x q_y - k_y q_x}{q\sqrt{k^2+m^2}}\right)\right], \quad (3.6)$$

$$K(\mathbf{r},t) = \sum_q \frac{e_a^2}{2\omega_q^3}(1 - e^{-i\omega_q t} e^{i\mathbf{q}\cdot\mathbf{r}}). \quad (3.7)$$

Performing the Fourier transform, the spectral function $A^{(1)}(\mathbf{k},\omega)$ is given by $A^{(1)}(\mathbf{k},\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,G^{(1)}(\mathbf{k},\omega)$. (For the connection with the weak coupling analysis, see Appendix B.)

So far the bare vertex function has been used for the computation. However, in the long-wavelength limit taking the bare vertex is not appropriate as manifestly seen by infrared divergence in $K(\mathbf{r},t)$. For the vertex part, random phase approximation is applied with respect to the longitudinal interaction term. The bare vertex is reduced by the factor of $1/(1-\pi_q)$, where

$$\pi_q = v_q \sum_k \mathrm{Tr}[G_k \gamma_0 G_{k+q}\gamma_0] = -\frac{m e_a^2}{\pi}\frac{1}{q^2} + O(1), \quad (3.8)$$

with $v_q = e_a^2/q^2$. For this computation, it is convenient to use the Euclidean formalism because the main contribution comes from π_q with $q=(\mathbf{q},0)$, where the Minkowskiian formalism leads to the same result.

Including the vertex correction the function $K(\mathbf{r},t)$ is

$$K(\mathbf{r},t) = \frac{e_a^2}{2}\int_0^\Lambda dq \frac{1}{q^2}\left(\frac{1}{1-\pi_q}\right)^2 [1 - \mathrm{e}^{-i\omega_q t} J_0(qr)], \quad (3.9)$$

with $J_0(x)$ the zeroth order of the Bessel function of the first kind. The ultraviolet cutoff Λ is introduced here because the wavelength of fluctuations is larger than the lattice constant. The integrand is expanded with respect to q, before the integration. The result is

$$K(\mathbf{r},t) \simeq iE_s t + \tfrac{1}{8}\Delta^2 r^2 + \tfrac{1}{4}\Delta^2 t^2, \quad (3.10)$$

where

$$E_s = \frac{e_a^2}{4}\log\left(\frac{\pi\Lambda^2}{e m e_a^2}\right), \quad (3.11)$$

$$\Delta^2 = \frac{e_a^2\Lambda}{2\pi}. \quad (3.12)$$

The first term in Eq. (3.9) represents the energy shift, E_s. The shape is changed to a broad Gaussian form by the subsequent terms as shown below. For the factor R_k, the analytic expression was not obtained. From a numerical computation I found that R_k is linear in k at $0<k<k_c$, with $k_c \simeq 1$, and reaches a saturated value of 0.2 for $k>k_0$. To approximate R_k, I took an approximate form of $R_k \simeq 0.2k$. In computing the Fourier transform, it is useful to note that the integration with respect to $\mathbf{r}$ and that with respect to t are carried out separately. For $k \ll \Delta$, I obtained

$$A^{(1)}(\mathbf{k},\omega) \simeq \frac{0.10}{\sqrt{8\pi^2}}\exp\left[-\frac{(\omega - E_k - E_s)^2}{\Delta^2}\right]. \quad (3.13)$$

Therefore, the Dirac fermion energy spectrum is shifted by E_s, which is associated with the energy gap of creating a hole, and is a Gaussian form with the width of Δ. For the estimation of these values, the spin wave velocity is assumed to be, $c_{sw} = 1.6J$. The mass term is evaluated as $m \simeq 1.3J$ from the ARPES experiments by fitting the dispersion near $(\pi/2,\pi/2)$. The gauge charge is simply taken from the factor of the Maxwellian term obtained by integrating out the Dirac fermions: $e_a^2 = 3\pi m$. (If the same calculation is carried out for Dirac fermions with $k>k_0$, then the gauge charge value is $e_a^2 \simeq 3\pi^2 k_0/4$, for $k_0 \gg m$. Therefore, the above choice is the minimum value for the gauge coupling.) Recalling the fact that c_{sw} is taken unity in the above calculation, I found $E_s \simeq 2J$ and $\Delta \simeq 3J$. This value of Δ is consistent with the above assumption about k. The weight of the Gaussian spectrum is ~ 0.05 for this value of Δ. Because of the spin degrees of freedom and the degenerate nodes, the weight is ~ 0.2 in total. From the value of E_s, one can get insight about the strength of the coupling. If the system is in the (weak) strong coupling, the value of E_s is expected to be large (small). The fact that E_s is on the order of the bandwidth suggests that the coupling is in the strong-coupling regime.

IV. DISCUSSION

In this paper, the spectral function of the quasiparticle in the π-flux state with phase fluctuations was computed within

the effective theory described by the QED_3. It was shown that the quasiparticle spectra become a broad Gaussian form with an energy shift due to the gauge field interaction. The estimated spectrum width is consistent with the experiments. The analysis suggests that the coupling to the gauge field is in the strong-coupling regime.

The formation of a Gaussian line shape is associated with emissions of many gauge photons by an excited hole. The situation is reminiscent of an optical emission in semiconductors. Here gauge photons are emitted instead of photons.

Since the model is based on the continuum approximation, the result is applicable to the quasiparticle excitations near $(\pm\pi/2, \pm\pi/2)$. However, the result is extended to other $\mathbf{k}$ points by formulating the theory on the lattice. Such a model is useful to study the change of the width of the spectra away from $(\pm\pi/2, \pm\pi/2)$.

As for the vertex correction, random phase approximation is applied with respect to the interaction arising from the longitudinal component of the gauge field. Of course, this is not the full vertex correction. In the long-wavelength limit, there are other intermediate processes. However, it is expected that dominant contribution is covered by the above vertex correction because the longitudinal component plays a major role in screening the gauge charge.

Now I comment on another QED_3 approach proposed by Franz *et al.*[16] In that theory, a Dirac fermion spectrum arises from Bogoliubov quasiparticle excitations around the nodes of a $d_{x^2-y^2}$-wave superconducting state. Phase fluctuations about the superconducting state are described by a U(1) gauge field, which is minimally coupled to the Dirac fermions. By contrast, the Dirac fermions discussed in this paper are associated with excitations about the antiferromagnetic long-range ordering phase. The U(1) gauge field describes the antiferromagnetic correlations. Thus, the physical context is quite different. The relationship between these two theories is not clear at present.

Finally, let me comment on vanishing quasiparticle spectra observed in the experiments[1,2] along the line from $(\pi/2, \pi/2)$ to (π, π) and that from $(\pi, 0)$ to (π, π). A similar behavior is also observed in the pseudogap phase,[17] that is, only a part of the Fermi surface is observed as an arc shape.[18] One might expect that damping effect coming from the coupling to the gauge field leads to the suppression of the quasiparticle peaks. However, it turns out that the inclusion of a slight hopping term in the π-flux state leads to the vanishing of the spectrum in the second magnetic Brillouin zone. I will discuss this matter in a future publication.

ACKNOWLEDGMENTS

I would like to thank T. Tohyama for helpful discussion. This work was supported by Grant-in-Aid for Young Scientists and the 21st Century COE "Center for Diversity and Universality in Physics" from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan. The numerical calculations were carried out in part on Altix3700 BX2 at YITP in Kyoto University.

APPENDIX A: DERIVATION OF QED_3 ACTION

In this appendix, I derive the QED_3 action as the effective theory for the $S=1/2$ two-dimensional Heisenberg antiferromagnet as follows:

$$H = J\sum_{\langle i,j\rangle} \mathbf{S}_i \cdot \mathbf{S}_j. \tag{A1}$$

A fermion representation is introduced for the spin 1/2: $\mathbf{S}_{j\mu}=f^\dagger_{j\alpha}\sigma_{\alpha\beta}f_{j\beta}/2$ $(\mu=x,y,z)$. σ^μ are the Pauli spin matrices. These fermions need to satisfy the constraint, $\Sigma_\alpha f^\dagger_{j\alpha}f_{j\alpha}=1$. Introducing Lagrange multipliers to take into accout the constraint, the Hamiltonian is

$$H = -\frac{1}{2}J\sum_{\langle i,j\rangle} f^\dagger_{i\alpha}f_{j\alpha}f^\dagger_{j\beta}f_{i\beta} + \sum_j \lambda_j(f^\dagger_{j\sigma}f_{j\sigma} - 2S), \tag{A2}$$

up to a constant term. The mean field taken in the π-flux state is $\chi_{ij}=\langle f^\dagger_{j\alpha}f_{i\alpha}\rangle\cdot h$, by choosing a suitable gauge.[19] Since the system is homogeneous, uniform χ_{ij} and λ_j are assumed: $\chi_1=\chi_{j+\hat{x},j}$, $\chi_2=\chi_{j,j-\hat{y}}$, $\chi_3=\chi_{j-\hat{x},j}$, and $\chi_4=\chi_{j,j+\hat{y}}$, with j residing at an even site. Numerically solving the mean field equations for $\chi_j(j=1,2,3,4)$, with setting $\lambda=0$, the π-flux state[10] is found in which $\chi_1\chi_2\chi_3\chi_4/|\chi_1\chi_2\chi_3\chi_4|=-1$.

The quasiparticle energy dispersion in the π-flux state is

$$E_k = \pm\frac{J}{2}|\chi_1 e^{-ik_x} + \chi_2^* e^{ik_y} + \chi_3 e^{ik_x} + \chi_4^* e^{-ik_x}|. \tag{A3}$$

$|E_k|$ has minima at $(\pm\pi/2, \pm\pi/2)$, and around these points the energy dispersion has the relativistic form. Introducing the even and odd site fields, $f_{ek\alpha}=(f_{k\alpha}+f_{k+Q,\alpha})/\sqrt{2}$ and $f_{ok\alpha}=(f_{k\alpha}-f_{k+Q,\alpha})/\sqrt{2}$ with $Q=(\pi,\pi)$, and expanding E_k around $(\pm\pi/2, \pi/2)$, the Hamiltonian is rewritten as

$$\begin{aligned} H \simeq J\sum_k{}' (f^\dagger_{e1k\alpha}\, f^\dagger_{o1k\alpha}) &\begin{pmatrix} 0 & -\chi_1^* k_x + \chi_2 k_y \\ -\chi_1 k_x + \chi_2^* k_y & 0 \end{pmatrix} \\ &\times \begin{pmatrix} f_{e1k\alpha} \\ f_{o1k\alpha} \end{pmatrix} + J\sum_k{}' (f^\dagger_{e2k\alpha}\, f^\dagger_{o2k\alpha}) \\ &\times \begin{pmatrix} 0 & \chi_1^* k_x + \chi_2 k_y \\ \chi_1 k_x + \chi_2^* k_y & 0 \end{pmatrix} \begin{pmatrix} f_{e2k\alpha} \\ f_{o2k\alpha} \end{pmatrix}. \end{aligned} \tag{A4}$$

Here indices 1 and 2 are introduced to denote the fields around $(\pi/2, \pi/2)$ and those around $(-\pi/2, \pi/2)$. The summation with respect to k is taken over the magnetic Brillouin zone: $|k_x \pm k_y| < \pi$. Choosing $\chi_1=\chi_3=i|\chi|$ and $\chi_2=\chi_4=|\chi|$, and setting $\psi^\dagger_{k\alpha}=(f^\dagger_{e1k\alpha}, f^\dagger_{o1k\alpha}, f^\dagger_{o2k\alpha}, f^\dagger_{e2k\alpha})$, the action is, in the continuum limit,

$$S = \int d^3x \bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x), \tag{A5}$$

where $\bar{\psi}=\psi^\dagger\gamma^0$ and the γ matrices are

$$\gamma^0 = \begin{pmatrix} \tau_3 & 0 \\ 0 & -\tau_3 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\tau_1 & 0 \\ 0 & -i\tau_1 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} i\tau_2 & 0 \\ 0 & -i\tau_2 \end{pmatrix}.$$

Phase fluctuations about the π-flux mean field state are included by the U(1) gauge field, a_μ, by replacing ∂_μ with $\partial_\mu - ia_\mu$. Integrating out the high-energy Dirac fermion fields with $k>k_0$ leads to the dynamics of the gauge field, which has the Maxwellian form. Numerically solving the Schwinger-Dyson equation, it is found that the self-energy has a nonzero mass.[12] Physically this mass is associated with

the presence of the staggered magnetization which is absent at the mean field level. From the variational Monte Carlo approach with the finite mass m, it is shown that the mean field energy improves by the inclusion of m.[20,21] The action (2.1) is obtained by including the mass term arising from the staggered magnetization.

APPENDIX B: WEAK-COUPLING ANALYSIS

In the weak-coupling regime, one may expand the factor $\exp[-K(\mathbf{r},t)]$ in Eq. (3.5) with respect to e_a^2 as follows:

$$\exp[-K(\mathbf{r},t)] = 1 - \sum_q \frac{e_a^2}{2\omega_q^3}(1 - \mathrm{e}^{-i\omega_q t}\mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}}) + \cdots . \quad \text{(B1)}$$

After the Fourier transform, we obtain, for the retarded Green's function,

$$G_k^{(1)}(\omega) = \frac{R_k}{\omega - E_k + i\delta} + \sum_q \frac{e_a^2}{2\omega_q^3}\left[\frac{R_{k+q}}{\omega - E_{k+q} - \omega_q + i\delta} - \frac{R_k}{\omega - E_k + i\delta}\right] + \cdots . \quad \text{(B2)}$$

The first term is the free Dirac fermion propagator with R_k the renormalization factor. The second term is the propagator with the self-energy correction of emitting single photon. Higher order terms are negligible in the weak-coupling regime. In that case, the line shape is of Lorentzian. While in the strong-coupling regime, it is necessary to sum up all orders of diagrams. However, it is a formidable task.

*morinari@yukawa.kyoto-u.ac.jp

[1] B. O. Wells, Z. X. Shen, A. Matsuura, D. M. King, M. A. Kastner, M. Greven, and R. J. Birgeneau, Phys. Rev. Lett. **74**, 964 (1995).
[2] S. LaRosa, I. Vobornik, F. Zwick, H. Berger, M. Grioni, G. Margaritondo, R. J. Kelley, M. Onellion, and A. Chubukov, Phys. Rev. B **56**, R525 (1997).
[3] F. Ronning, C. Kim, D. L. Feng, D. S. Marshall, A. G. Loeser, L. L. Miller, J. N. Eckstein, I. Bozovic, and Z. X. Shen, Science **282**, 2067 (1998).
[4] K. M. Shen, F. Ronning, D. H. Lu, W. S. Lee, N. J. C. Ingle, W. Meevasana, F. Baumberger, A. Damascelli, N. P. Armitage, L. L. Miller, Y. Kohsaka, M. Asuma, M. Takano, H. Takagi, and Z. X. Shen, Phys. Rev. Lett. **93**, 267002 (2004).
[5] C. L. Kane, P. A. Lee, and N. Read, Phys. Rev. B **39**, 6880 (1989).
[6] M. Brunner, F. F. Assaad, and A. Muramatsu, Phys. Rev. B **62**, 15480 (2000).
[7] T. Tohyama and S. Maekawa, Supercond. Sci. Technol. **13**, R17 (2000).
[8] J. Bala, A. M. Oles, and J. Zaanen, Phys. Rev. B **52**, 4597 (1995).
[9] A. S. Mishchenko and N. Nagaosa, Phys. Rev. Lett. **93**, 036402 (2004).
[10] I. Affleck and J. B. Marston, Phys. Rev. B **37**, 3774 (1988).
[11] R. B. Laughlin, Phys. Rev. Lett. **79**, 1726 (1997).
[12] D. H. Kim and P. A. Lee, Ann. Phys. (N.Y.) **272**, 130 (1999).
[13] T. C. Hsu, Phys. Rev. B **41**, 11379 (1990).
[14] G. Mahan, *Many-Particle Physics*, 3rd ed. (Plenum, New York, 2000).
[15] T. K. Ng, Phys. Rev. B **63**, 245109 (2001).
[16] M. Franz, Z. Tesanovic, and O. Vafek, Phys. Rev. B **66**, 054535 (2002).
[17] A. Damascelli, Z. Hussain, and Z. X. Shen, Rev. Mod. Phys. **75**, 473 (2003).
[18] M. R. Norman, H. Ding, M. Randeria, J. C. Campuzano, T. Yokoya, T. Takeuchi, T. Takahashi, T. Mochiku, K. Kadowaki, P. Guptasarma, and D. G. Hinks, Nature (London) **392**, 157 (1998).
[19] P. A. Lee, N. Nagaosa, and X. G. Wen, Rev. Mod. Phys. **78**, 17 (2006).
[20] C. Gros, Ann. Phys. (N.Y.) **189**, 53 (1989).
[21] T. K. Lee and S. P. Feng, Phys. Rev. B **38**, 11809 (1988).